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Title

Learning and Understanding Division: A Study in Educational Neuroscience

Authors

Stephen R Campbell

Faculty of Education
Simon Fraser University
8888 University Drive
Burnaby, BC, V5A 1S6
Canada

Email: sencael@sfu.ca
Phone: 1-778-782-3630
Fax: 1-778-782-3203
Web: www.engrammetron.net
Email: sencael@sfu.ca

O. Arda Cimen, & K. Handscomb

Faculty of Education
Simon Fraser University

Abstract

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Running head

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AERA 2009 Proposal: Brain, Neuroscience, and Education SIG

Stephen R. Campbell, O. Arda Cimen, & K. Handscomb
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Learning and Understanding Division: A Study in Educational Neuroscience

Abstract

This pilot study applies methods of educational neuroscience to investigate and improve preservice teachers' learning and understanding of whole number and rational number division and connections between the two using a Computer Enhanced Mathematics Learning Environment (CEMLE).

Objectives or purposes:

A widely recognized concern in elementary school mathematics education is that teachers' understanding of the mathematical curricular content generally appears to be quite fragmented, sparsely connected, and procedurally oriented (Ball, 1990; Campbell, 2002; Simon, 1993). Moreover, it has been well documented that middle school learners' understanding of connections between whole, fractional, and decimal numbers can be quite problematic (e.g., Mack, 1995; Markovits & Sowder, 1991; Greer, 1987).

Campbell (2002; 2006) identifies a potential source of these difficulties as related to the conceptual and procedural evolution of arithmetic division, suggesting that whole number and rational number division are not being adequately distinguished, neither conceptually nor procedurally, and that connections between them are not adequately considered.

A computer-enhanced mathematics learning environment (CEMLE) has been designed and developed (Campbell & Fonthal, 2000), called *DivFact*, enabling learners to explore the conceptual and procedural connections between whole number and rational number division. Many questions have emerged pertaining to the design, implementation and efficacy of this CEMLE, and how to study its use (e.g., Campbell, 2000; 2003a).

A general question guiding this study is: How do CEMLEs contribute to, or inhibit, mathematical cognition and learning? The specific goal in this research is to investigate aspects of preservice teachers' use of DivFact from different theoretical perspectives pertaining to computer-enhanced graphical and symbolic representations of whole number and rational number division.

Specific objectives include determining what kinds of learning styles are manifest through the use of DivFact; how effective is DivFact in helping learners to understand division; and in what ways can the design of this CEMLE be improved. An important counterpart of this research is developing and applying methods of educational neuroscience toward this purpose.

Perspectives or theoretical framework:

The nature of computer assisted learning and its potential role in mathematical cognition and learning has long been a focal point of research in mathematics education. Relatively new is the emergence of two distinct views about the nature of such tools, and their role in teaching and learning: 1) the *cognitive view*, oriented toward the content of learning; and 2) the *situative view*, placing emphasis on contexts associated with learning.

More recently there has emerged a third paradigm, referred to as the *embodied view* (Varela, Thompson & Rosch, 1991), which designates the lived experience of human embodiment emerging from and situated within the world as the primary vehicle for cognition, affect, behavior, and social interaction.

Other counterparts to this study are comparative assessments of cognitivist, situativist, and embodied theoretical perspectives. We argue for an embodied perspective as most appropriate approach to acquiring, analyzing, and interpreting the data acquired in this study (Campbell & the ENL Group, 2007; Campbell & Handscomb, 2007, April).

To meet our aforementioned objectives, we require conceptual and theoretical frameworks of mathematical content and learning mathematics with CEMLEs, combined with methodologies suitable for the detailed study of learning with computer-enhanced mathematics learning environments. CEMLEs like DivFact can be defined by three general characteristics:

1. they are used for teaching, learning, or exploring various aspects of mathematics
2. they typically use graphical and/or symbolic modes of representation
3. they can be made widely accessible to learners over the internet

Whole number and rational number division

The first goal of this study is to investigate the effectiveness of DivFact in helping discern teachers' learning styles, and independently of their orientation toward graphical or symbolic representations, to enable them as learners to understand differences between whole-number division and rational-number division, and procedural and conceptual connections between them.[1]

Whole number division rests on the division theorem, a fundamental theorem of number theory (Campbell, 2002; 2006). Accordingly, for any whole number A and any natural number D, referred to as dividend and divisor, respectively, there exist unique whole numbers Q and R, referred to as quotient and remainder, respectively, such that

$A = QD + R$, where $D \neq 0$, and $0 \leq R < D$

The division theorem is applied iteratively, for instance, in long division. The units constituting whole numbers cannot be divided, hence the need for a remainder.

[1] Caps and small letters indicate whole and rational number variables respectively.

Rational number division rests on an algebraic property of the field of rational numbers. Accordingly, if d is any non-zero rational number, then d has a multiplicative inverse, d^{-1}, within the field of rational numbers. Therefore, if a is any rational number, then there is a rational number q such that

$a = qd$, where $q = a\,d^{-1}$

A major difference between whole number division and rational number division is that the former involves a whole number remainder (possibly zero) and in the latter, because units comprising rational numbers can be divided, i.e., denominated into equal parts, entails a fractional component (possibly zero) that is often rendered as a decimal.

Typically, conceptual difficulties occur in understanding relations between the remainder and the fractional component. Such difficulties become readily evident when learners utilize calculators to perform division when whole number division is the required operation. Consider, the following simple example:

$13/2 = 6.5$

A learner may identify either 0.5 or 5 as the remainder. Methods to determine the remainder by means of a calculator are not immediately evident for many learners, who have difficulty with these procedures, even if connections between them are understood. Such difficulties may be compounded if the fractional component is a non-terminating decimal. DivFact (Fig. 1) has been designed to help learners explore connections and overcome difficulties between and with whole and rational number division.

Cognitive, situative, and embodied views of learning with CEMLEs

A task of increasing importance for researchers in mathematics education is to better understand the nature and role of CEMLEs in learning. This task is of secondary concern to proponents of the cognitivist view, whose primary focus is on the cognitive content of the learner, usually in relation to the subject content to be learned (e.g., Anderson, 1983; Winograd & Flores, 1986). Cognitivist concepts and language readily apply to CEMLEs, as cognitivists often tend to cast cognition in terms of computer metaphors.

There has also been growing interest in situated cognition and learning: the notion that cognitive development inextricably involves the context of the learner, interactions with others, activities the learner is engaged in, and the various tools involved in and goals for engaging those activities (e.g., Latour, 1987; Lave, 1988; Walkerdine, 1988). Some take this growing interest as a major paradigm shift in cognitive science, one on par with, if not exceeding in importance, the shift from behaviourism to cognitivism (Kirshner & Whitson, 1997). The situative view rightly emphasizes the importance of context in learning, and CEMLEs are becoming an increasingly important and prevalent aspect of that context.

Some theorists and researchers suggest (e.g., Bruner, 1997; Kirshner & Whitson, 1997), that the cognitive and situative views are complementary, but there has been some debate in this regard (e.g., Anderson, Reder, & Simon, 1996, 1997; Greeno, 1997). One outcome of this debate has been greater scrutiny on mathematical reasoning, representation, and on the role of tools and inscriptions in the emergence of mathematical understanding (e.g., Campbell, 2003a,b; Cobb, 2002; Roth & Bowen, 2001; Roth & McGinn, 1998).

Differences between cognitivist and situative views follow from differences in units of analyses in the study and understanding of cognition and learning. The unit of analysis in the cognitive view is the *individual*. The unit of analysis in the situative view is more the *context* from which learners emerge and are embedded. Ongoing debate between these perspectives typically concerns priorities given to the individual verses social interactions, and the respective roles of thought and language.

From a cognitive view, a measure of effectiveness for a given representation—be it symbolic or graphical—is how well the representation models the structure it is meant to represent. The more "transparent" the representation of a mathematical concept is, the better (Lesh, Post & Behr, 1987): "A transparent representation would have no more nor less meaning than the idea(s) or structure(s) they represent" (ibid., p. 56).

From a situative view, knowing is "knowing your way around in an environment and knowing how to use its resources" (Greeno, 1991, p. 175). Following Wertsch (1991), Meira (1998, p. 122) divides tools into two general categories: "signs (semiotic systems) and technical tools (instrumental artifacts)." Accordingly, it is evident that arithmetic and algebraic symbols fall into the former category, while graphical mathematical tools such as straight-edge and compass fall in the latter category.

CEMLEs, being both hardware-software, symbolic-graphic, and representational-functional machines, typically blend and blur these cognitivist and situativist categories. New ways of thinking about signs and tools are in order, and this is where an embodied view may provide greater coherence and some novel insights.

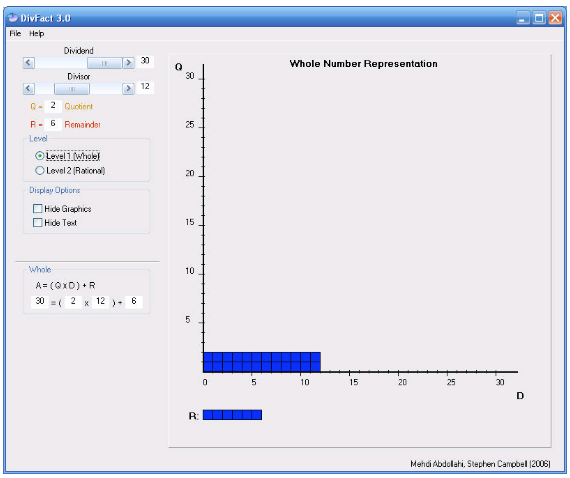
Embodied views, like the cognitive view, trace their origins in part to cybernetics (Gardner, 1985; McCulloch, 1965; Weiner, 1948; Winograd & Flores, 1986). Predominantly driven by metaphors such as control, communications, and feedback, cybernetics has largely been transmuted into more contemporary studies concerning design, interaction, and self-regulated learning. From the formal perspective of general system theory (Bertalanffy, 1968), such concepts, their precursors and kin, apply equally well across biological, psychological, sociological, and technological phenomena. This suggests analytical grounds for relating cognitivist and situative views.

It is hypothesized that an embodied approach can serve well to unite the respective foci of the cognitivist and situativist views (Campbell, 2003b; Campbell & Dawson, 1995; Núñez, Edwards, & Matos, 1999), especially learning with CEMLEs (Campbell, 2000), and thereby warrants empirical study from that perspective.

Methods, techniques, or modes of inquiry:

At least 24 and up to 30 preservice elementary school teachers seeking their teaching certificates from a professional development program are being recruited for this study. Participants provide a suite of psychometric data pertaining to health, demographics, mathematics and related mathematical anxieties.

This study applies methods of educational neuroscience (Campbell & the ENL Group, 2007) to investigate elementary preservice teachers' learning and understanding of whole number and rational number division and connections between the two. Participants are prepared, pretested, and provided with a 15 to 20 minute time frame to explore the DivFact CEMLE (Fig. 1) in both whole number and rational number modes.



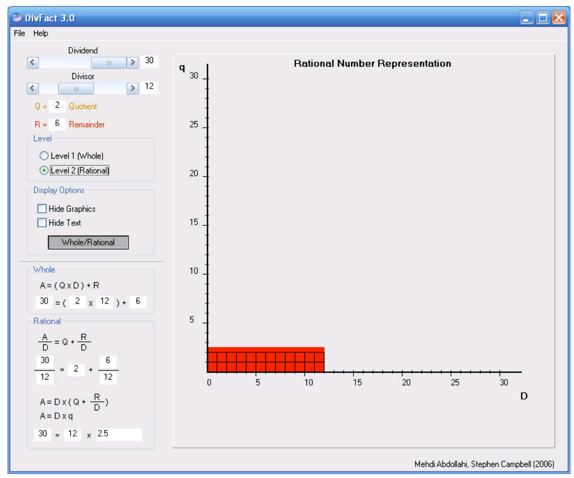
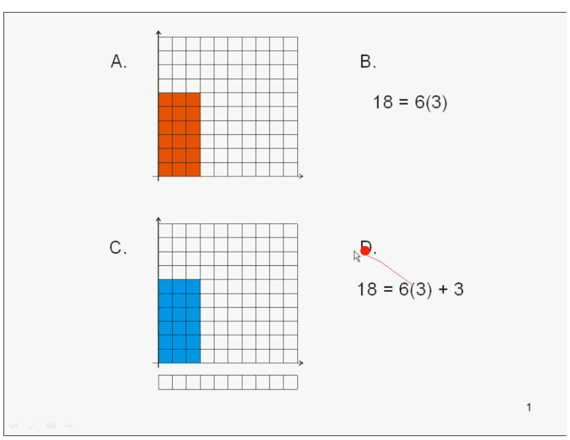


Figure 1a: Whole number division

Figure 1b: Rational number division

Participants are then post-tested with a suite of 36 'odd ball' slides (Fig 2), where the task is to identify the graph or symbolic equation that is inconsistent with the other three.



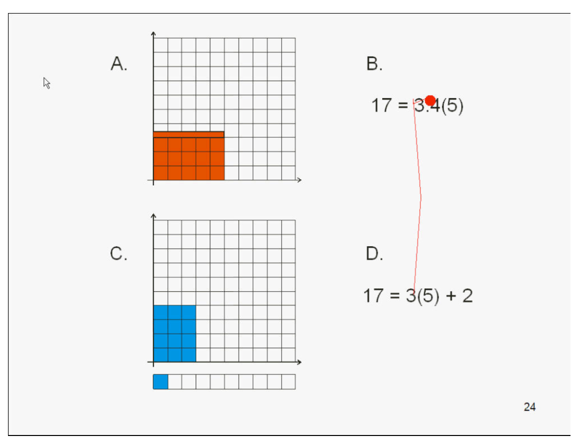


Figure 2a: A whole number test slide

Figure 2b: A rational number test slide

Data sources or evidence:

The red lines terminating in red balls in Figure 2 indicate the participant's eye movements and gaze location leading up to the moment in time from the video from which these sample screen grabs were taken (viz., for the sake of illustration here).

Using electroencephalography (EEG), participants are prepared with scalp electrodes to record the voltage potentials generated by their brains during both the exploration and the assessment activities. Figure 3 illustrates the kinds of behavioral and physiological data collected, integrated, and time synchronized for analysis in course of this study.

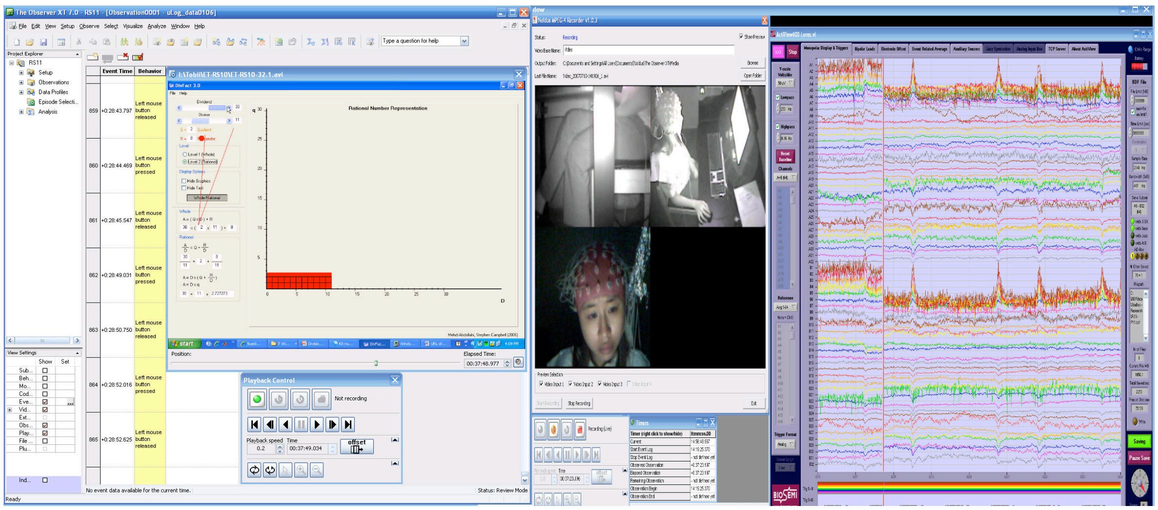


Figure 3: An integrated time synchronized data set illustrating eye-tracking and EEG

Data analysis, results and conclusions

Data are currently being analyzed, and preliminary results will be discussed at the meeting. In accord with the embodied view, changes in our participants' subjective lived experience must manifest in some objective embodied manner. Identifying such correlates will provide unprecedented new insights into learning and understanding.

Informed by the eye-tracking data and drawing upon results from cognitive neuroscience (e.g., Dehaene, 1996), we seek such correlates in the EEG data with cognitive functions of perception and reflection, counting, calculating, estimating, graphical and symbolic modalities of representation, and identifying differences between each of these.

In accord with the conference theme, applying methods of educational neuroscience afford a quantum leap in observational methods typically and traditionally available to educational researchers. They are to audiovisual methods much as audiovisual methods are to hand written field notes. By applying these methods, this study in embodied cognition aims to gain deeper insight into preservice teachers' understandings and aversions toward the learning and understanding of division.

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